

**MAS/118 Differential Equations**  
**Marking scheme**

**Problem 1** (similar to coursework)

a)

$$\underbrace{y = \int \frac{1}{(x-2)(x-3)} dx}_{\boxed{2P}} = \underbrace{\ln|x-3| - \ln|x-2|}_{\boxed{3P}} + \underbrace{C}_{\boxed{1P}}$$

$$-1 = y(0) = \ln 3 - \ln 2 + C \quad \boxed{2P} \quad \Rightarrow C = \ln 2 - \ln 3 - 1 \quad \boxed{1P}$$

b)

$$e^{-y} dy = x/(x^2 + 1) dx \quad \boxed{2P} \quad \Rightarrow \underbrace{-e^{-y}}_{\boxed{1P}} = \underbrace{\ln(x^2 + 1)/2}_{\boxed{2P}} + \underbrace{C}_{\boxed{1P}}$$

$$-1 = -e^{-y(0)} = \ln(1)/2 + C \quad \boxed{2P} \quad \Rightarrow y = -\ln(1 - \ln(x^2 + 1)/2) \quad \boxed{2P}$$

c)

$$\underbrace{y' = \frac{3x^2 + 1}{2\sqrt{x^3 + x}}}_{\boxed{3P}} = \underbrace{3 \frac{x^2 + 1}{2\sqrt{x^3 + x}}}_{\boxed{3P}} - \frac{1}{y} = \frac{3y}{2x} - \frac{1}{y} \quad \boxed{3P}$$

**Problem 2** (similar to coursework)

a)

$$A(x) = - \int \left( \frac{1+x}{1+x} - \frac{1}{1+x} \right) dx = -x + \ln|1+x| \quad \boxed{2P}$$

$$u(x) = e^{-A(x)} = e^x/(1+x) \quad \boxed{2P}$$

$$u(x)y(x) = \int u(x)(1+x)e^{-x} dx \quad \boxed{2P}$$

$$= \int dx = x + C \quad \boxed{2P}$$

$$y(x) = (1+x)e^{-x}(x+C) \quad \boxed{1P}$$

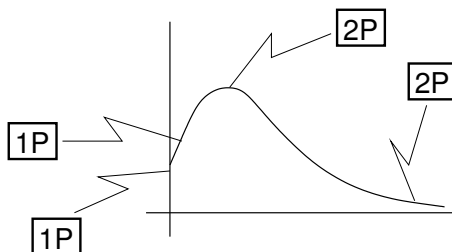
$$1 = y(0) = (1+0)1(0+C) = C \quad \boxed{1P}$$

b)

$$y'(0) = -0y(0) + 1 = 1 \quad \boxed{2P}$$

$$(1+x)^2 e^{-x} \rightarrow 0 \text{ as } x \rightarrow \infty \quad \boxed{2P}, \text{ thus } y(x) \rightarrow 0 \quad \boxed{2P}.$$

c)



**Problem 3** (similar to coursework)

a)

$$2\lambda^2 + 3\lambda + 1 = 0 \quad \boxed{1P} \Rightarrow \lambda^{(A)} = -1/2 \quad \boxed{1P} \quad \lambda^{(B)} = -1 \quad \boxed{1P}$$

$$y = C^{(A)} e^{-x/2} + C^{(B)} e^{-x} \quad \boxed{2P}$$

b)

$$y_p = A(x)e^{-x/2} + B(x)e^{-x} \quad \boxed{1P}$$

$$A(x) = \int \frac{e^{x/2} e^{-x}/2}{-1/2 + 1} dx \quad B(x) = \int \frac{e^x e^{-x}/2}{-1 + 1/2} dx \quad \boxed{3P}$$

$$A(x) = -2e^{-x/2} \quad B(x) = -x \quad \boxed{2P}$$

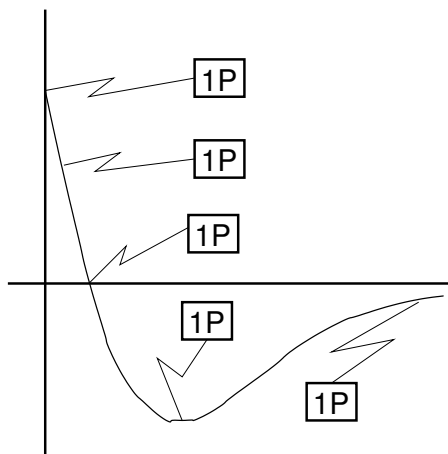
$$y_p = -2e^{-x} - xe^{-x} \quad \boxed{1P}$$

$$y = C^{(A)} e^{-x/2} + C^{(B)} e^{-x} - xe^{-x} \quad \boxed{1P}$$

c)

$$1 = y(0) = C^{(A)} + C^{(B)} \quad \boxed{1P} \quad -2 = y'(0) = -C^{(A)}/2 - C^{(B)} - 1 \quad \boxed{1P}$$

$$C^{(A)} = 0, \quad C^{(B)} = 1 \quad \boxed{1P}$$



**Problem 4** (similar to coursework)

a)

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 4 \\ -2 & -5 \end{pmatrix} \Rightarrow 0 = \det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = \lambda^2 + 4\lambda + 3 = (\lambda + 1)(\lambda + 3) \quad \boxed{2P}$$

$$\lambda^{(A)} = -1 \quad \boxed{1P}, \quad \lambda^{(B)} = -3 \quad \boxed{1P}$$

$$\begin{pmatrix} 2 & 4 \\ -2 & -4 \end{pmatrix} \underline{u}^{(A)} = 0 \quad \boxed{1P} \Rightarrow \underline{u}^{(A)} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \boxed{1P}$$

$$\begin{pmatrix} 4 & 4 \\ -2 & -2 \end{pmatrix} \underline{u}^{(B)} = 0 \quad \boxed{1P} \Rightarrow \underline{u}^{(B)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \boxed{1P}$$

b)

$$\underline{y}(x) = C^{(A)} \begin{pmatrix} 2 \\ -1 \end{pmatrix} \exp(-x) + C^{(B)} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \exp(-3x) \quad \boxed{2P}$$

c)

$$1 = 2C^{(A)} + C^{(B)} \quad \boxed{1P}$$

$$0 = -C^{(A)} - C^{(B)} \quad \boxed{1P}$$

$$= -C^{(A)} + 2C^{(A)} - 1 \quad \boxed{1P}$$

$$C^{(A)} = 1 \quad C^{(B)} = -1 \quad \boxed{2P}$$

$$y_1(t) = 2 \exp(-x) - \exp(-3x) \quad \boxed{1P}$$

$$y_2(t) = -\exp(-x) + \exp(-3x) \quad \boxed{1P}$$

d)

$$y_1'(0) = \underbrace{y_1(0) + 4y_2(0)}_{\boxed{1P}} = 1 \quad \boxed{1P}$$

$$y_2'(0) = \underbrace{-2y_1(0) - 5y_2(0)}_{\boxed{1P}} = -2 \quad \boxed{1P}$$

e)

